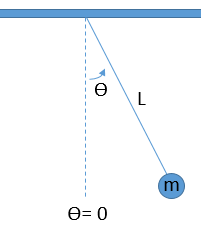
**Damped Driven Pendulum**

**Learning Goals:**

* To build a computational model of a simple pendulum using the Euler-Cromer algorithm.
* To produce graphs of the angular displacement and angular velocity of the pendulum as a function of time from the results of the computational model.
* To assess the accuracy of the computational results of the model.
* To identify the limitations of the small angle approximation for the pendulum.



**PART 1: A simple pendulum**

Consider a simple pendulum with a point mass *m* at the end of a string of length *L*. Performing a torque analysis to find the angular acceleration of the system, we have:

(1)

This equation can be solved analytically only using the small angle approximation, sin(θ) ≈ θ, in which case θ(t) = A cos(ωt+φ), with .

1. Solve equation (1) numerically using the Euler-Cromer method:

ωn = ωn-1 + αn-1 Δt

θn = θn-1 + ωn Δt

With the following parameters:

Mass m = 1 kg

Length L = 1 m

Gravitational acceleration g = 9.81 m/s2

And the following initial conditions:

Initial angular displacement A = 0.1 radians

Initial angular velocity ω0 = 0 radians/s

Time step Δt = 1 s

2. Plot ϴ(t) vs t.

3. Change Δt to smaller values until the solution converges (i.e. it does not change significantly as you make Δt smaller).

4. On the same figure as (2), plot the solution for the small-angle approximation:

θ(t) = A cos(ωt+φ), with

5. Now change the initial value of the angular displacement, *A*, to larger values. For what angle is the small angle approximation no longer a good approximation? How did you determine this?

**PART 2: A damped simple pendulum**

Friction and air resistance can be included as a “damping” term, which is proportional to the angular velocity:

Where *b* is a constant that characterizes the damping effects.

1. Modify your angular acceleration to include this term, use *b* = 0.2 kg m2/s, then plot ϴ(t) vs t.

2. How many oscillations will it take for the amplitude to decrease to 1/*e* of its initial value?

**PART 3: A damped and driven simple pendulum**

The pendulum can be driven by a sinusoidal torque, which will add a third term (this one positive) to the angular acceleration:

1. Modify your angular acceleration to include this term. Use *ωd* = 1 rad/s and τd = 0.3 Nm, then plot ϴ(t) vs t for at least five oscillations.

2. Using the small angle approximation, the natural angular frequency of this system is . What happens to the amplitude of the oscillations when the driving frequency is each of the following frequencies?

1. = 0.5
2. = 0.9
3. =
4. = 1.1

**PART 4**

If you have completed PARTS 1 - 3, you may choose to do some further analysis of the system. Some suggestions:

* Amplitude vs. frequency (no damping/driving)
* Amplitude vs. driving frequency with damping (small amplitude)
* Shift of resonant frequency in driven pendulum as amplitude increases (nonlinearities)